

# **Tolerance Design of Ballast Resistance for Multi-string LED Driver**

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## Keywords

« Lighting », « Electronic ballast», « Design », « Diode », « Device simulation»

### Abstract

This paper investigates a novel tolerance design method for a resistive light-emitting-diode (LED) driver with multi-string configuration in mass production. Considering a trade-off among the current uniformity, the current accuracy in LEDs, and the loss dissipation in the driver circuit, a bus voltage and a series resistor are identified as two design parameters regardless of production spreads both in the LED string voltage and in the ballast resistance. By combining discrete optimization technique with Monte-Carlo method, this paper proposes an effective algorithm to design the circuit parameters. Comparison with the conventional design by simulation and hardware tests shows that the suggested scheme provides more reliability and flexibility in practical situation. Moreover, utilizing only datasheet values eliminates the need for I-V curve extraction in the design process.

## Introduction

Nowadays, multi-string light-emitting-diode (LED) arrays are popular in lighting and display applications because they can be easily constructed without increasing the supply voltage too much. However, LEDs inherently have production spread in their forward voltage and string structure having multiple LEDs in series accumulates the deviation and deteriorates the situation. Therefore, when driven by direct voltage sources, currents in parallel LED strings are imbalanced as shown in Fig. 1 because of difference between the I-V characteristic curves of the LED strings. In mass production, the current mismatch becomes worse.

The main function of the LED driver is to provide nearly equal current in each string to ensure uniform brightness even when there is a production spread in LED forward voltage, and consequently to increase the lifespan of the LEDs. Many researchers have investigated current balancing methods, including linear and switching power converter driver circuits. A dedicated dc/dc converter solution [1]-[5], magnetic balancing techniques [6]-[8], and capacitive balancing circuits [9]-[14] are achievements of those research.

Meanwhile, a resistive balancing circuit shown in Fig. 2 is mostly adopted for small power applications such as decoration lamps or liquid crystal display (LCD) backlights of mobile display devices, in which the number of series-connected LED packages in a string is small and the LED current is low[15]. This is because of the low cost and extreme simplicity of the circuit. Having no additional controller and a minimum part count enhances the reliability of the whole system. In this circuit configuration, a front-end dc/dc converter provides a regulated bus voltage, and a series ballast resistor in each string balances the LED current.





Fig. 2: LED current mismatch in direct parallel driving

To design the balancing resistor value in this circuit, several methods can be used. For example, a design equation for the target LED current is solved first, and a nominal resistor value close to the calculated value may be chosen as in [16]. However, this method does not provide the optimal design because it provides no design guidelines for the bus voltage and there is no consideration for the production spread of the series resistor. A more elaborate method based on a graphical technique has been proposed in [17] which utilizes extreme I-V curves of LED string to find the optimal value of the series ballast resistor and the bus voltage as shown in Fig. 3. This method is very simple and powerful, but two extreme curves must be obtained from the LED package vendor before applying this method, which is not common in actual situations. Moreover, the calculated resistance value should be rounded off to the nearest nominal EIA (Electronic Industries Alliance) standard values such as E12, E24, and so on. Therefore, it is difficult to design the nominal value of the resistor in the presence of production spreads both in the LED forward voltage and in the series resistor. Moreover, the performance indices such as current accuracy, current uniformity, and power dissipation cannot be dealt with at the same time.

In this paper, a statistical design algorithm for the series resistor and bus voltage is proposed. This method combines discrete optimization with Monte-Carlo analysis to optimize the LED current distribution, such as the uniformity between the parallel branches and the accuracy of the LED current, and, at the same time, loss dissipation in the driver circuit. The suggested design method is verified by hardware test results, and performance comparisons with the conventional scheme are performed.



Fig. 1 : Resistive LED balancing circuit





Fig. 3 : Conventional design method

# **Problem definition**

#### Statistical distribution in the circuit components

In mass production, each LED package shows a slightly different forward voltage because it has been picked at random from among samples having the same nominal value. Because several LEDs are assembled in series into an LED string, the overall forward voltage of the LED string also shows a probabilistic distribution and if the number of series connections is large, the distribution approaches Gaussian according to the central limit theorem [18].

From this fact, the LED string voltage,  $V_D$ , can be regarded as a random variable that has a truncated normal distribution with the upper limit and lower limit specified from the manufacturer's datasheet. In practice, we may not know the exact value of  $V_D$ , since the LEDs may have been picked at random. For each LED string, the string voltage will have a value between those two extreme values.

Similar situation also happens to the ballast resistance,  $R_S$ . Because resistance also has production tolerance, we may not know the real value of  $R_S$  in advance since any resistor may have been picked in a random manner during mass production, so the actual value is slightly different from the original design value. Moreover, the design value should be chosen among the nominal values,  $R_{S,nom}$  which belong to the EIA standard number given by

$$R_{S,nom} = 10^{l + \frac{m}{E}} \ (0 \le m < E, \ l > 0)$$
<sup>(1)</sup>

where both 1 and m are integer and E is the EIA standard number. That is, the design value is discretized number. In this research, it is assumed that every  $R_s$  in each branch is designed with the same nominal value.

#### **Performance indices**

A regulated bus voltage,  $V_s$ , and a series resistor,  $R_s$ , in each string are parameters to design in order to achieve the high performance index regardless of the production spread. As for the performance index of the LED driver, accuracy as well as uniformity of the LED string currents should be considered. Accuracy is the degree of closeness to the target current specification, whereas uniformity is the degree to which the current in each branch is equalized.

The LED current in each string should be equalized and maintain the target current regardless of production spread, and is calculated as

$$I_D = \frac{V_S - V_D}{R_S}.$$

On the other hand, the loss dissipation in the series resistor is represented as

$$P_{L} = I_{D}^{2} R_{S} = \frac{(V_{S} - V_{D})^{2}}{R_{S}}.$$
(3)

Therefore, in order to make the current independent of the spread in  $V_D$ , the ballast resistor and the bus voltage should be increased, which means the accuracy and the uniformity should be sacrificed in order to suppress high loss dissipation in the balancing resistors. Component tolerances in the resistors may make the current accuracy even worse. It is evident that there is a design trade-off among performance indices.

### **Proposed Design Method**

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#### Parameter Space, Region of Tolerance, and Region of Acceptance

In order to consider the problems mentioned above, it is natural to assume that both  $R_S$  and  $V_D$  have normal distributions with upper and lower limits, namely truncated normal distribution, and they are statistically uncorrelated between each other. This assumption does not restrict the generality of the proposed algorithm. If we construct a parameter space with axes of series resistance,  $R_S$ , and LED string voltage,  $V_D$ , as in Fig. 4(a), a single point in this space describes an implementation of a parallel branch having the corresponding resistor value and LED string voltage. Therefore, if  $R_S$  and  $V_D$  have tolerances of  $t_{RS}$  and  $t_{VD}$ , an implementation of the circuit may lie within a rectangular region of tolerance (ROT) and the occurrences will be scattered as in Fig. 4(a), and any additional parallel branch can also be regarded as another occurrence of the  $R_S$ - $V_D$  pair. For example, a two-channel LED circuit will have two points on the ROT at the same time. Therefore, any difference in LED currents evaluated at those two points results in a current mismatch between the LED strings.

The center point of the ROT represents the nominal value of the LED string voltage,  $V_{D,nom}$ , and the nominal series resistor value,  $R_{S,nom}$ . While  $V_{D,nom}$  is pre-determined by LED datasheet,  $R_{S,nom}$  is a design variable compliant with EIA standard and the center of the ROT exists only in a discretized location given by (1). Likewise, while the tolerance of the LED string voltage,  $t_{VD}$ , is the difference between the upper and lower limit values in the datasheet, the tolerance of the resistance,  $t_{RS}$ , is given



Fig. 4 Parameter space in V<sub>D</sub> vs. R<sub>D</sub> (a) ROT (b) ROA

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by the distance between the two neighboring nominal values and thus is dependent on the nominal resistor value. The standard deviations  $\sigma_{VD}$  and  $\sigma_{RS}$  can be easily obtained as one-sixth of the corresponding tolerance values, which is the convention in statistical analysis with the Gaussian assumption [19][20].

Let us consider the LED current at a certain implementation point  $(R_s, V_D)$  in the parameter space. With the upper and lower limit specification on the target LED current given by (2), a constraint is

$$I_{D,MIN} \le \frac{V_S - V_D}{R_S} \le I_{D,MAX},\tag{4}$$

and will be mapped to the parameter space as the area between the straight lines that pass through the y-intercept of  $V_S$ , and the gradients are  $I_{D,MIN}$  and  $I_{D,MAX}$  as in Fig. 4(b).

The specification also requires the power dissipation given by (3) to be less than the upper limit as in the following equation:

$$\frac{\left(V_{S} - V_{D}\right)^{2}}{R_{S}} \le P_{R,MAX}$$
(5)

and this condition will be transformed into the upper area of the parabola that passes through the y-intercept of  $V_s$  in parameter space of Fig. 4(b).

The overlapping area between (4) and (5) is the region of acceptability (ROA) of the linear LED driver and is denoted as the shaded area in Fig. 4(b). If a point in the ROT which represents a branch of mass-produced circuits lies within the ROA, then the performance of the branch satisfies the specifications even with the production spread.

#### **Algorithm and Design Strategies**

Thus the design problem is to determine two design variables, the bus voltage,  $V_S$ , and the nominal series resistance,  $R_{S,nom}$ , to minimize a penalty function in the presence of truncated Gaussian distribution in both  $V_D$  and  $R_S$ . The mathematical description is then defined as follows.

Given:

$$V_{Dnom}, \sigma_{V_D}, E, N, V_{S,max}$$
Minimize  $P(V_S, R_{S,nom})$ 
Subject to :
$$V_{D,i} \sim N(V_{Dnom}, \sigma_{V_D}) \text{ and } R_{S,i} \sim N(R_{Snom}, \sigma_{R_S}) \text{ for } i = 1,...,N$$

$$(V_{D,nom}, R_{S,nom}) \in ROA$$

$$0 < V_S \leq V_{S,max}, \text{ continuous}$$

$$0 \leq m < E, l > 0, \text{ integer}$$
(6)

Because there are discrete parameters to be optimized, this problem requires discrete optimization techniques. Among various methods, exhaustive enumeration [21] that searches for the optimal solution of mathematical model for all possible combinations of the discrete variables is known to be the simplest and thus is adopted in this paper.

Figure 5 shows the flow chart for the proposed algorithm. From the datasheet values, the nominal and tolerance values of  $V_D$  are obtained first. Throughout the algorithm,  $V_S$  and  $R_{S,nom}$  are searched for in the following manner. Once a bus voltage is selected, the location of the ROA is determined because the bus voltage will fix the y-intercept value of the ROA. For each bus voltage, every possible  $R_{S,nom}$  is tried to locate a new ROT in the parameter space. Then, Monte-Carlo analysis is performed to check the performance of the circuit at that design point. The algorithm generates a large number of



samples, N, sprayed in the corresponding ROT, which simulates a mass production situation and evaluates the elements functions defined by

$$S = \sqrt{\frac{1}{N} \sum_{i=1}^{N} \left( \frac{V_{S} - V_{D,i}}{R_{S,i}} - \frac{1}{N} \sum_{i=1}^{N} \frac{V_{S} - V_{D,i}}{R_{S,i}} \right)^{2}},$$
(7)

$$E = \left| \frac{1}{N} \sum_{i}^{N} \frac{V_{S} - V_{D,i}}{R_{S,i}} - I_{D,TARGET} \right|,$$
(8)

$$L = \frac{1}{N} \sum_{i=1}^{N} \frac{\left(V_{s} - V_{D,i}\right)^{2}}{R_{s,i}} \,.$$
(9)

The element functions S and E represent the uniformity and the accuracy of the LED current, and L describes the driver circuit loss. These functions are calculated by the standard deviation of the LED current, the average errors from the target current, and the average power loss in the series resistor. For



Fig. 5: Proposed algorithm



each element function, the smaller value indicates better performance. For example, a smaller S indicates a more uniform current distribution between parallel strings, and a smaller E indicates that the average LED current is closer to the target current. After completion of the evaluation, another ROT is chosen and the sample generation and evaluation procedure are performed again. When  $R_{S,nom}$  reaches its final value, the scanning of the ROT is stopped and a new bus voltage is tried. This process repeats over the entire range of  $V_S$ .

At the end of the algorithm, the penalty function, P, for each design candidate is calculated by summing up the element functions S, E, and L and normalizing with its maximum value that has been found. It is defined by

$$P = w_1 \cdot S/S_{MAX} + w_2 \cdot E/E_{MAX} + w_3 \cdot L/L_{MAX}$$
(10)

$$0 \le w_1, w_2, w_3 \le l, \quad w_1 + w_2 + w_3 = l \tag{11}$$

where  $w_1$ ,  $w_2$ , and  $w_3$  are weighting coefficients and the subscript 'MAX' denote the maximum value of each element. Using different weighting coefficients, a different design strategy can be set. Finally, the optimal design point that has the minimum penalty function evaluation is chosen among the candidates.

### Hardware verifications

For a prototype circuit, we constructed a dual-channel passive LED driver with three 0.3 W LED packages in one string (PCL- D9WCZ11SC provided by Power Lightec Inc.). The recommended target current was 60 mA and E24 (+/- 5%) values for the series resistor were considered.

We tested 10 LED string samples numbered from 1 to 10, and extracted the I-V curve for each from the direct voltage driving measurements. Table I shows the experimental V-I characteristics of the samples. Samples 3 and 9 show the extreme characteristics having a 150 mV difference at the same driving current of 60 mA.

Starting from a specification of  $I_{D,MIN} = 58$  mA,  $I_{D,MAX} = 62$  mA and applying the conventional design approach in Fig. 3, a load line was drawn by two extreme characteristic curves of Samples 3 and 9, and  $R_s$  was calculated as 12.5 ohms and then rounded off to 13 ohms which is the nearest possible E24 value. By the x-intercept of the load line, the bus voltage is determined to be 10.3 V.

To apply the proposed method, another condition,  $P_{R,MAX} = 0.5$  W, was added to the constraint. Though the lower and upper limit of the LED forward voltage can be obtained directly from the LED datasheet, the two extreme measured value in Table I were used instead for comparison purposes.  $V_{D,nom}$  is determined as the center value and  $\sigma_{VD}$  were calculated as one sixth of the tolerance by truncated normal distribution assumption.

Among the LED string samples, the extreme string voltages of 9.43 V (Sample 9) and 9.57 V (Sample 3) were regarded as the lower limit and the upper limit for  $V_D$  distribution, and  $V_{D,nom}$  and  $t_{VD}$  were calculated as 9.50 V and 0.14 V, respectively. The nominal value of the series resistor was found in the E24 numbers, and 10% of the selected nominal value determined  $t_{VR}$ . The number of Monte-Carlo simulations, N, for each ROT candidate was set to 1,000. The proposed algorithm was programmed in a MATLAB m-file script. For each possible design point, 1,000 samples were sprayed inside the ROT to simulate mass production, and the element functions (7)-(9) were evaluated and stored. Finally, the penalty function is calculated by (10) and design point showing the minimum P value was selected as the final solution. We chose four different design strategies and the weighting coefficients ( $w_1, w_2, w_3$ ) were set to (0.8,0.1,0.1), (0.1,0.8,0.1), (0.1,0.1,0.8), and (0.33,0.33,0.33) for

Table I: Measured forward voltage in LED strings (I<sub>D</sub>=60mA)

Sample No.	#1	#2	#3	#4	#5	#6	#7	#8	#9	#10
$V_F(V)$	9.53	9.50	9.57	9.51	9.54	9.50	9.52	9.53	9.43	9.51

design strategies M1, M2, M3, and M4.

The simulated performances for the various design strategies are summarized in Table II where M0 refers to the conventional design result. Every solution shows a different value for the element functions S, E, and L. By applying strategy M1, M2, or M4, a lower S value can be achieved, which means the differences in LED current from one sample to another were smaller than those obtained by the conventional design. In other words, the LED currents in the parallel branches were uniformly balanced. Observations also indicate that the proposed design allows for smaller values of E than those obtained by the conventional design, which means the average LED currents were closer to the target current in mass production.

However, there is a trade-off between LED current performance and power loss. In particular, the solution obtained by strategy M1 shows superior performance in both S and E, however it shows an extremely high value of L. The situation improved with M2, though it still shows a high loss. Considering the overall performances, M3 resulted in the smallest power loss with a reasonable current distribution. M4 also resulted in better current performance with a slightly increased power dissipation.

To obtain hardware test results, it is ideal to run a mass production in order to evaluate the performance of each design results. However, this is time-consuming and is not cost-effective. In this paper, as a more reasonable verification method, an edge test was performed for the prototype hardware. Fig. 6 shows the parameter combinations for the tests. At first, the series resistances,  $R_{S1}$  and  $R_{S2}$ , were intentionally selected to be equal to the nominal values for each strategy. String Samples 2 and 6 were used for Branches 1 and 2, and thus  $V_{D1}$  and  $V_{D2}$  had forward voltages of 9.495 V, which is exactly the nominal value as shown in Fig. 6(a). Secondly,  $V_{D1}$  and  $V_{D2}$  remained at 9.495 V, but  $R_{S1}$  and  $R_{S2}$  were trimmed to be -5% and +5% of the nominal value, respectively (Fig. 6(b)). Finally, the series resistors were selected as the nominal value and the LED string voltages,  $V_{D1}$  and  $V_{D2}$  had extreme values, i.e., Sample 3 (9.57 V @ 60 mA) for Branch 1, and Sample 9 (9.43 V @ 60 mA) for Branch 2 as shown in Fig. 6(c). For each case, the LED current in each branch was measured and the corresponding penalty function is evaluated.

To estimate the overall performance, the hardware results from the edge tests should be weighted by the corresponding probability of occurrence, which can be obtained using the two-variable Gaussian probability density function without correlation [22],

$$pdf(R_{S}, V_{D}) = \frac{1}{2\pi\pi_{R_{S}}\sigma_{V_{D}}} exp\left[-\frac{(R_{S} - R_{S,nom})^{2}}{2\sigma_{R_{S}}^{2}} - \frac{(V_{D} - V_{D,nom})^{2}}{2\sigma_{V_{D}}^{2}}\right],$$
(12)

and the overall performances were compensated using the following formula,

$$Overall Penalty = pdf(R_{S,nom}, V_{D,nom})^{4} \times \{penalties \text{ in edge test } (a)\}$$
  
+  $pdf(R_{S,lower}, V_{D,nom})^{2} pdf(R_{S,upper}, V_{D,nom})^{2} \times \{penalties \text{ in edge test } (b)\}$   
+  $pdf(R_{S,nom}, V_{D,lower})^{2} pdf(R_{S,nom}, V_{D,upper})^{2} \times \{penalties \text{ in edge test } (c)\}$ (13)

Design method		Solut	tion	Penalties (relative ratio)				
		Vs	R <sub>S,nom</sub>	S	Е	L		
M0		10.3	13	1.00	1.00	1.00		
	M1	14.0	75	0.47	0.05	5.40		
Dropogod	M2	11.3	30	0.61	0.09	2.20		
Proposed	M3	10.1	10	1.20	0.30	0.73		
	M4	10.7	20	0.73	0.18	1.47		

**Table II: Design results and performances** 





Fig. 6: Parameter combination for edge tests

Table III: Hardware edge test results

Design method		Solutions		Standa	rd dev.	Penalties (relative ratio)			
		Vs	R <sub>S,nom</sub>	$\sigma_{RS}$	$\sigma_{VD}$	S	Е	L	
M0		10.3	13	0.217	0.0233	1.00	1.00	1.00	
Proposed	M1	14.0	75	1.250	0.0233	0.23	0.40	5.60	
	M2	11.3	30	0.500	0.0233	0.50	0.64	2.20	
	M3	10.1	10	0.167	0.0233	0.97	0.88	0.80	
	M4	10.7	20	0.333	0.0233	0.73	0.71	1.40	

and the results are shown in Table III. Comparison with the simulation results in Table II shows that the proposed method provides better performance with design flexibility than the conventional design method.

## Conclusion

In this paper, an effective design algorithm for a resistive current-balancing LED driver was proposed by combining discrete optimization with Monte-Carlo analysis. It simultaneously optimizes LED current distribution and loss dissipation in the driver circuit. Besides considering mass production tolerances, the proposed method also provides more reliable design and design flexibility using different weighting strategies for the penalty function. Moreover, contrary to the conventional method, utilizing only datasheet values eliminates the need for I-V curve extraction for the LEDs.

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